Econ G25T: Open Economy Macroeconomics

Problem and Discussion Questions #1

ANSWER KEY (in bold)

Please note that the answer key is not comprehensive and may not be fully rigorous.
请注意，答案并不全面，也可能不够严谨。

1. Suppose a homogenous good is sold in the domestic market by two firms, one domestic and one foreign. Demand in the domestic market is given by a linear demand curve: 假设一个同质商品由两家公司在国内市场销售，一家是国内的，一家是国外的。国内市场需求以**线性需求曲线**表示：

$$Q = a - bP, \tag{1}$$

where Q is the total output of the good sold in the domestic market and P its price. There are two firms, one domestic and one foreign, which sell the good in the domestic market. Sales of the domestic (foreign) firm in the domestic market are given by q (q^*), so $Q = q + q^*$. Each firm maximises profits, taking as given the output of the other firm. (This is the standard Cournot assumption.)

The (constant) marginal cost of producing a unit of output is W for the domestic firm and W^* (in foreign currency terms) for the foreign firm. Also, E is the exchange rate (domestic currency price of one unit of foreign currency) and is treated by the firms as exogenous.

其中 Q 为国内市场销售商品的总产出，P 为其价格。有两家公司，一家国内的，一家国外的，在**国内市场**销售产品。国内企业在国内市场的销售额用 q 表示，国外企业在国内市场的销售额用 q* 表示，因此 Q = q + q*。每个公司都以另一个公司的产出为条件，使利润最大化。(这是标准的古诺假设)

生产一单位产品的(恒定)边际成本对国内企业是 W，对外国企业是 W*(以外币计算)。

E 也是汇率(单位外币的本国货币价格)，被企业视为**外生变量**。

Profits for the domestic firm are hence 国内企业因此获得的利润为：

$$\Pi = (P - W)q, \tag{2}$$

and for the foreign firm they are 对于外国公司来说也是如此：

$$\Pi^* = (P / E - W^*)q^*, \tag{3}$$

(a) Each firm chooses its output to maximise its profits. Give the two first-order conditions for profit maximisation with respect to choice of output levels. Note: each firm needs to take into

account the effects of its actions on *P.* 每个公司都选择自己的产出来最大化利润。就产量水平的选择给出利润最大化的两个一阶条件。注:每个公司都需要考虑其行为对 P 的影响。

From the demand curve, we have $P = (a - Q)/b = (a - q - q^*)/b$. **Substituting this into equation (2), we derive the following expression for the domestic firm's maximand:** 根据需求曲线，我们有 P = (a - Q)/b = (a - q - q*)/b。将其代入式(2)，得到国内企业的最大值表达式如下:

$$ \tag{4} $$

The first order condition for maximization with respect to *q* is as follows (applying the product rule):
q 最大化的一阶条件如下(应用乘积法则):

$$ \tag{5} $$

2

Similarly, the foreign firm's maximand can be written 同样，外国公司的最大利润也可以写出来

$$(6)$$

The first-order condition is
一阶条件为：

$$(7)$$

(b) Using your results in (a), derive each firm's reaction function – that is, an expression for the firm's output in terms of the other firm's output, the parameters and other variables. 利用(a)中的结果，推导出每个公司的反应函数——也就是说，用另一个公司的产量、参数和其他变量表示该公司的产量。

Re-arranging equation (5) gives 重新整理式(5)得

$$q = \{a - q* - bW\} / 2. \qquad (8)$$

This is the domestic firm's reaction function. 这就是国内企业的反应函数。

Carrying out the maximisation, we derive the following reaction function for the foreign firm: 为了实现最大化，我们为外国企业推导出如下的反应函数：

$$q* = \{a - q - bEW*\} / 2. \qquad (9)$$

(c) Depict the reaction functions diagrammatically, in a diagram with q on one axis and $q*$ on the other. What happens to each firm's reaction function when the exchange rate changes? 以图表形式描述反应函数，q 在一个轴上，q*在另一个轴上。当汇率变化时，每个公司的反应函数会发生什么变化？

See below for the reaction function diagram. The curve labelled *DD* is the domestic firm's reaction function and that labelled *FF* is the foreign firm's reaction function. 下面是反应函数图。 标为 DD 的曲线为**国内企业**的反应函数，标为 FF 的曲线为**国外企业**的反应函数。所以对于国内企业来说，反应函数为：q*近似等于-2q。对于国外企业来说，反应函数为：q*近似等于 -0.5q。所以才有下图所示的两种直线。

From the equations of the reaction functions, it should be clear that the domestic firm's reaction function does not change when the exchange rate changes, whereas the foreign

firm's reaction function shifts inwards. This is shown as the movement from *FF* to *F'F'* in the diagram below. 从反应函数方程可以看出，国内企业的反应函数不随汇率变化而变化，而国外企业的反应函数则向内变化。如下图所示，这是从 FF 到 F ' F '的移动（随着 E 增大，FF 向下移动到 F′F′）。
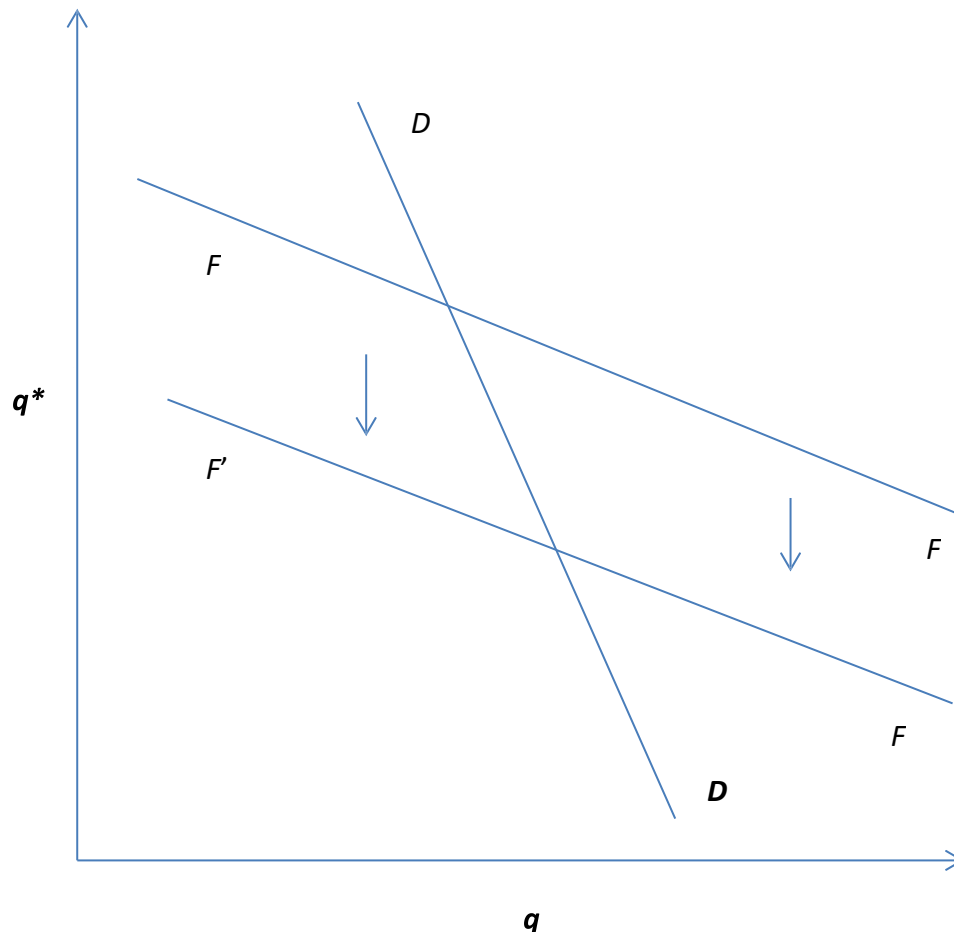
DD：国内企业反应函数　　FF：国外企业反应函数

Cournot Oligopoly Diagram – Effect of Depreciation of Exchange Rate 古诺寡头图——汇率贬值的影响

(d) Using the reaction functions, derive expressions for each individual firm's output, for total output and the price of the good in terms of the parameters, the exchange rate, and *W* and *W**.利用反应函数，推导出每一家公司的产量、总产出和商品的价格与参数、汇率、W 和 W*的关系。

Substituting for $q*$ from (9) into (8), we obtain 将(9)中的 q*代入(8)中，得到

$$2q = a - \{a - q - bEW\}/2 - bW. \tag{10}$$

Solving, we obtain

$$q = \frac{a - 2bW + bEW^*}{3} \tag{11}$$

Similarly, we obtain the following expression for *q**:
同理，我们得到 q*的表达式为:

$$\tag{12}$$

$$q^* = \frac{a + bW - 2bEW^*}{3}$$

Total output is hence given by 因此，总产出为

$$Q = q + q^* = \frac{2a - b(W + EW^*)}{3}. \tag{13}$$

Substituting this into the demand curve, we obtain the following expression for the price:
将其代入需求曲线，得到价格的表达式如下:

$$P = \frac{a + b(W + EW^*)}{3b}. \tag{14}$$

(e) Derive an expression for exchange-rate pass-through. What can be said about its magnitude? 推导汇率传递的表达式。关于它的大小我们能说些什么呢?

From (14) we can derive the following expression for exchange rate pass through: 由(14)我 们可以推导出以下汇率传递的表达式: ERPT 汇率传递，即 Exchange-rate Pass-through，本 质上是商品价格关于汇率的弹性，即汇率 E 变化 1%所带来的商品价格 P 变化的百分比。

$$\mathrm{ERPT} \equiv \frac{E}{P}\frac{dP}{dE} = \frac{EW^*}{3P} \tag{15}$$

It must be the case that *P/E > W** **or else the foreign firm would not enter the market – it will only supply goods to the market if it makes a positive profit. It follows, therefore, that ERPT in this case must be less than 1/3. 它必须是 P/E > W*的情况，否则，其他外国公司将不会 选择进入该国市场，因为 W*是外国公司的以它们外国的货币所表示的单位商品的成本，P/E 是外 国公司以它们外国的货币所表示的单位商品的价格。如果 P/E<W*，则外国企业会做赔本买卖， 因此外国公司铁定不会选择进入该国市场。当且仅当 P/E>W*，即单位商品售价大于其成本时，**

此时这笔买卖的利润才为正，只有利润为正时，外国企业才会选择在该国市场供应商品。因此，在这种情况下，也就是在 P/E > W*，也即 W*<P/E 这种情况下，必定有 ERPT 小于 1/3。

2.   Suppose the following equation governs the behaviour of the real exchange rate (q_t): 假设实际汇率(qt)的行为为:

$$q_t = \rho q_{t-1} + \beta, \text{with } \rho \leq 1.$$

Note that, for simplicity, but unrealistically, we assume there is no disturbance term. 注意，为了简单起见，我们假设没有扰动项存在，尽管这是不切实际的假设。

(i) Under what condition does the dynamic system have a steady state? 动态系统在什么条件下处于稳态?

ρ **must be less than 1 for there to be a steady state. 必须小于 1 才能有一个稳定的状态**

(ii) If a steady state exists, what is the steady-state value of q_t?
如果存在稳态，qt 的稳态值是多少?

The steady state value of q_t ($\overline{q}$) solves the equation $\overline{q} = \rho\overline{q} + \beta$. So $\overline{q} = \beta/(1-\rho)$.

(iii) (Assume the condition for the existence of a steady state is satisfied.) Suppose that for all $t < 0$, the real exchange rate is at its steady-state level, but at time $t = 0$, β increases to β'. Derive an expression for the time path that q_t takes in response to this shock. What can be said about the adjustment path the real exchange rate takes from time 0 onwards? Draw a diagram to illustrate.

（iii）（假设满足稳态的条件）假设对于所有 t <0，实际汇率处于稳态水平，但是在时间 t = 0 时，β 增加到 β′。 请你推导出 qt 响应这种冲击的时间路径表达式。 从时间 0 开始，实际汇率所需的调整路径是什么？ 画一个图来说明。

We can derive the following expressions: 我们可以推导出以下表达式:

$$q_{-1} = \beta/(1-\rho),$$
$$q_0 = \rho\beta/(1-\rho) + \beta',$$
$$q_1 = \rho^2\beta/(1-\rho) + \rho\beta' + \beta',$$
$$q_2 = \rho^3\beta/(1-\rho) + \rho^2\beta' + \rho\beta' + \beta', \text{ etc}$$

A pattern quickly establishes itself. The general expression for q_t is 模式很快就建立起来了。qt 的一般表达式是

$$q_t = \rho^{t+1}\beta/(1-\rho) + (1+\rho+\rho^2+...+\rho^t)\beta' =$$
$$\frac{\rho^{t+1}\beta}{1-\rho} + \frac{(1-\rho^{t+1})\beta'}{1-\rho} \tag{16}$$

In the limit (i.e. as t tends to infinity) the first term in this expression goes to zero and the second term converges to $\beta'/(1-\rho)$, which is the new steady state value of q. What

happens every period is that the real exchange rate adjusts to eliminate a fraction of the difference between the previous value of the real exchange rate and its new steady state value. Things may be clearer if we take a numerical example. Suppose $\beta = 1$, $\rho = 1/2$ and $\beta' = 2$, so the original steady-state value of q is 2 and the new steady-state value 4. Then it is easily calculated that the time path of q_t is as follows (starting with its value at time t -1): 2, 3, 3.5, 3.75, 3.8875, etc. It is easy to draw this.

在极限（即随着 t 趋于无穷大）中，该表达式的第一项变为零，并且第二项收敛于 $\beta'/(1-\rho)$，它是 q 的新稳态值。 在每个周期里都会发生的是，实际汇率会进行调整，以消除实际汇率的先前值与其新的稳态值之间的差值的一小部分。 如果我们举一个数字实例，事情可能会更清楚。 假设 $\beta = 1$, $\rho = 1/2$ and $\beta' = 2$，因此 q 的原始稳态值为 2，新的稳态值为

4。然后可以很容易地计算出 qt 的时间路径如下（从时间 t-1 的值开始）：2，3、3.5、3.75、3.8875 等。绘制起来很容易。

(iv) What is the 'half-life' of the adjustment process described above if (a) $\rho = 0.99$ or if (b) instead $\rho = 0.97$? Note: 'half-life' is defined as the time expected for the real exchange rate to move half the way back to its steady-state level, so if it is initially 10% above its steady state level, it's the time it takes for the real exchange rate to move to a 5% overvaluation.

如果（a）$\rho = 0.99$ 或（b）$\rho = 0.97$，上述调整过程的"半衰期"是什么？

注意："半衰期"定义为实际汇率回到其稳态水平的一半所需的时间，因此，如果最初比稳态水平高出 10%，则该时间就是 实际汇率变化到比稳态水平高 5%水平所花费的时间。

The difference between the new steady state value of *q* and q_0 is easily calculated to be $\dfrac{\rho}{1-\rho}(\beta' - \beta).$ **The difference between the new steady state value of *q* and q_t can be similarly**

calculated (using (16)) to be $\dfrac{(\beta' - \beta)\rho^{t+1}}{1-\rho}$ **. The half-life, *T*, hence solves the equation 新的稳态值 q 与 q_0 之间的差值很容易计算出来，为 。q 和 qt 的新稳态值之间的差异可以通过类似公式 (16) 这样的方法计算出来，为 。半衰期（T）就是该方程的解。**

This clearly reduces to $\rho^T = 0.5$ **. So we have** $T = \log(0.5)/\log\rho$. **This means that when**

$\rho = 0.99$ ***T* is approximately 69 and when** $\rho = 0.97$**, *T* is approximately 23. If the length of a period is one month, then the first case corresponds to a half-life of about five years and nine months, whereas the second corresponds to a half-life of one year and eleven months.**

这意味着当ρ=0.99 时，半衰期 T 大约为 69，而当 ρ = 0.97 时，T 大约为 23。如果期间的长度为一个月，则第一种情况对应于大约 5 年零 9 个月的半衰期，而第二种情况对应的半衰期为 1 年零 11 个月。

3. Suppose the price of widgets in the UK is P_W and their euro price is $P_W{}^*$. All the usual assumptions underlying the law of one price are assumed to hold, except that there is a transport cost of *T* for transporting a widget between the UK and the euro area. What can be said about the relationship between the two prices? (I.e. derive a relationship which replaces the law of one price equation.) 假设英国的小部件价格是 P_W，它们的欧元价格是 $P_W{}^*$。

除了在英国和欧元区之间运输一件产品的运输成本是 T 之外，所有基于一价定律的通常假设都是成立的。关于这两个价格之间的关系，我们可以得到些什么呢?(换句话说，推导出一种关系来代替一价定律的方程式)

The relationship is $SP_W{}^*-T \leq P_W \leq SP_W{}^*+T$.

If the first inequality is not satisfied, it would be profitable for an arbitrageur to buy widgets in the UK, ship them to Germany and make a profit after paying the transport cost. If the second inequality is not satisfied, then it will be profitable for an arbitrageur to buy the goods in Germany and sell them in the UK, making a profit after paying the transport cost. The essential point is that two inequality constraints replace one equation.

如果第一个不等式不被满足，套利者可以在英国购买小部件，然后把它们运到德国，在支付了运输成本后再获利。如果不满足第二个不等式，那么套利者在德国购买商品，然后在英国出售，在支付运输成本后获利。要点是两个不等式约束代替了一个等式。

Is it true that the higher the transport costs of transporting a good, the greater the divergence from the law of one price must be? 运输货物的运输成本越高，与一价定律的偏离就越大，这对吗？

No, if transport costs for one good are higher than that for another good, it doesn't follow that the divergence from the LOOP <u>must</u> be greater for that good. The divergence <u>may</u> be greater, but this does not mean that it must be higher. Put another way, higher transport costs may increase the <u>maximum</u> divergence possible from the LOOP, they may not increase the <u>actual</u> divergence. 不，如果一种商品的运输成本高于另一种商品的运输成本，则并不能因此认为该商品与 LOOP 的差异必须更大。 差异可能更大，但这并不意味着它必须更高。 换句话说，较高的运输成本可能会增加与 LOOP 之间的最大差异，但可能不会增加实际差异。